Q. Using method of separation of variables

Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$; given $u(0,y) = 3e^{-y}$

Sol^n. Given

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \quad \text{—①}$$

Let the solution of eq^n ① be

$u(x,y) = X(x) \cdot Y(y)$ $\quad u = XY$

∴ eq^n ① becomes

$$X'Y = 4XY' \Rightarrow \frac{X'}{X} = \frac{4Y'}{Y} = K$$

Solving $\frac{X'}{X} = K$

on integration $\log X = Kx + \log C_1$

$\Rightarrow \log X = Kx \log e + \log C_1$

$= \log e^{Kx} + \log C_1$

$= \log C_1 e^{Kx}$

$\Rightarrow X = c_1 e^{kx}$

Solving $\frac{Y'}{Y} = k_1$

on integration $\log Y = k_1 y + \log c_2$

$\Rightarrow \log Y = k_1 y \log e + \log c_2$

$= \log e^{ky/4} + \log c_2$

$= \log c_2 e^{ky/4}$

$\Rightarrow Y = c_2 e^{ky/4}$

$\therefore u(x,y) = c_1 e^{kx} \cdot c_2 e^{ky/4}$

$= c_1 c_2 e^{k(x + y/4)}$

At $x = 0$, $u(0,y) = c_1 c_2 e^{ky/4}$

Also $u(0,y) = 8e^{-3y}$ (given)

on comparing both eq^n

$c_1 c_2 = 8$, $\frac{k}{4} = -3 \Rightarrow k = -12$

The req'd. solution is

$u(x,y) = 8e^{-12(x + y/4}$

$= 8e^{-12\left(\frac{4x+y}{4}\right)}$

$= 8e^{-3(4x+y)}$ #