

Q. using method of separation of variables

Solve $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$; given $u(0,y) = 3e^{-2y}$

Solⁿ. Given

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$$

Let the solution of eqⁿ (1) be

$$u(x,y) = x(x) \cdot Y(y) \quad u = xy$$

eqⁿ (1) becomes

$$x'Y = 4xY' \Rightarrow \frac{x'}{x} = \frac{4Y'}{Y} = K$$

Solving $\frac{x'}{x} = K$

on integration $\log x = Kx + \log C_1$

$$\Rightarrow \log x = Kx \log e + \log C_1$$

$$= \log e^{Kx} + \log C_1$$

$$= \log C_1 e^{Kx}$$

	2019		2019		2019
Mon	31	3	10	17	24
Tue		4	11	18	25
Wed		5	12	19	26
Thu		6	13	20	27
Fri		7	14	21	28
Sat		8	15	22	29
Sun		9	16	23	30

$$\Rightarrow x = c_1 e^{kx}$$

Solving $\frac{y'}{y} = \frac{k}{4}$

on integration $\log y = \frac{k}{4}y + \log c_2$

$$\Rightarrow \log y = \frac{k}{4}y \log e + \log c_2$$

$$= \log e^{ky/4} + \log c_2$$

$$= \log c_2 e^{ky/4}$$

$$\Rightarrow y = c_2 e^{ky/4}$$

$$\therefore u(x, y) = c_1 e^{kx} \cdot c_2 e^{ky/4}$$

$$= c_1 c_2 e^{k(x + y/4)}$$

$$\text{At } x=0, u(0, y) = c_1 c_2 e^{ky/4}$$

$$\text{Also } u(0, y) = 8e^{-3y} \text{ (given)}$$

on comparing both eqⁿ

$$c_1 c_2 = 8, \frac{k}{4} = -3 \Rightarrow k = -12$$

The reqd. solution is

$$u(x, y) = 8e^{-12(x + y/4)}$$

$$= 8e^{-12 \left(\frac{4x + y}{4} \right)}$$

$$= 8e^{-3(4x + y)} \quad \text{A}$$

2010	JULY				2010	AUGUST					
Mon	5	12	19	26	Mon	30	2	9	16	23	
Tue	6	13	20	27	Tue	31	3	10	17	24	
Wed	7	14	21	28	Wed		4	11	18	25	
Thu	1	8	15	22	Thu		5	12	19	26	
Fri	2	9	16	23	Fri		6	13	20	27	
Sat	3	10	17	24	Sat		7	14	21	28	
Sun	4	11	18	25	Sun		1	8	15	22	29